Balanced scoring method for multiple-mark questions

Keywords: multiple-choice questions, multiple-mark questions, scoring methods

Abstract: Advantages and disadvantages of a learning assessment based on multiple-choice questions (MCQs) are a long and widely discussed issue in the scientific community. However, in practice this type of questions is very popular due to the possibility of automatic evaluation and scoring. Consequently, an important research question is to exploiting the strengths and mitigate the weaknesses of MCQs. In this work we discuss one particularly important issue of MCQs, namely methods for scoring results in the case, when the MCQ has several correct alternatives (multiple-mark questions, MMQs). We propose a general approach and mathematical model to score MMQs, that aims at recognizing guessing while at the same time resulting in a balanced score. In our approach conventional MCQs are viewed as a particular case of multiple-mark questions, thus, the formulas can be applied to tests mixing MCQs and MMQs. The rational of our approach is that scoring should be based on the guessing level of the question. Our approach can be added as an option, or even as a replacement for manual penalization. We show that our scoring method outperforms existing methods and demonstrate that with synthetic and real experiments.

1 Introduction

Advantages and disadvantages of a learning assessment based on multiple-choice questions (MCQs) are a long and widely discussed issue in the scientific community. However, in practice this type of questions is very popular due to the possibility of automatic evaluation and scoring (Farthing et al., 1998). Consequently, an important research question is to exploit the strengths and mitigate the weaknesses of MCQs. Some systems (e.g. Moodle) allow teachers to create MCQs with multiple correct options. This type of questions we will call multiple-mark questions (MMQs), to distinguish them from the conventional MCQs, where there is always only one correct option. Multiple-mark questions were already recommended by Cronbach (Cronbach, 1941). Other research (Ripkey and Case, 1996; Pomplun and Omar, 1997; Hohensinn and Kubinger, 2011) considers MMQs to be more reliable, when compare them with conventional MCQs. However, even though the advantages of MMQs are meanwhile widely accepted, up to our knowledge there are no balanced methods for scoring multiple-mark questions available to date.

One possible approach to score the MMQs is to use dichotomous scoring system. The dichotomous scoring awards the constant amount of points, when the question is answered correctly and zero points in a case of any mistake. However, the partial scoring is preferable to the dichotomous, especially in case of MMQs. (Ripkey and Case, 1996; Jiao et al., 2012; Bauer et al., 2011; Ben-Simon et al., 1997)

The second possible approach is to use the methods, developed for scoring the multiple true-false questions (MTFs). However, despite the possibility to convert the MMQs into MTFs, the studies (Cronbach, 1941; Dressel and Schmid, 1953) show the differences between two formats. Moreover, the researches mentioned above named the following disadvantages of MTF questions compared to MMQs:

- The multiple true-false format “clouds” from the learners the possibility of marking several options as true.
- The level of reliability in multiple true-false questions is not equal for true and false answers.
- The multiple true-false format requires more resources to store the answers.

In the paper we show that the differences prevent the applying methods developed for the MTFs to the MMQs scoring.

Another possible approach is to use the penalties, similarly to the paper-based assessment where the teacher can analyze the student answers and decide how much points she deserves. The method was proposed by Serlin (Serlin and Kaiser, 1978). For example, in Moodle a teacher has to determine what penalty applies for choosing each distractor. However, this work is an additional, unpopular burden for teachers, since not required in paper-based tests. Instead of asking the teacher, some systems calculate the penalties

[https://moodle.org/](https://moodle.org/)
automatically. However, computer-based assessment opens additional possibilities to guess, for example choosing all options. Often the scoring algorithms do not take into account such ways of guessing.

Consequently, we are facing the challenge to find a scoring method, that is able to recognize and properly penalize guessing. Previously proposed algorithms suffer from imbalance and skewness as we show in Section 3.

The task to find the scoring method can be divided into two steps:

1. Find a method to determine points for the correctly marked options.
2. Find a method to determine the penalty for the incorrectly marked options.

For the first part a reasonable approach was proposed by Ripkey (Ripkey and Case, 1996). Thus our research aims to provide a method for the second part (determining penalties). We propose a general approach and a mathematical model, that takes into account the most common ways of guessing and behaves balanced at the same time.

Our concept is based on the assumption, that scoring can be based on the guessing level of the question. Each question is associated with a difficulty to guess a (partially) correct answer. To accommodate the difficulty level of guessing in the scoring method, we propose to determine the penalty only when a student marks more options, than the actual number of correct ones. We argue that our approach can be added as an option, or even as a replacement of manual designation of penalties. We claim that our algorithm behaves better, than existing ones and prove that with both synthetic and real experiments. In our approach conventional MCQs are viewed as a particular case of multiple-mark questions, thus, the formulas can be applied to the tests mixed of MCQs and MMQs. As the scoring of conventional MCQs is a trivial task, we do not consider such type of questions in our experiments.

The paper is structured as follows: First, we present the terminology we use. Then we discuss existing algorithms for scoring MMQs, which we have found in the research literature and real applications. After that we describe our approach on conceptual and mathematical levels. Finally we show and discuss the results of synthetic and real-life experiments.

2 Terminology

In the following we define the key concepts building the basis for our MMQ scoring method:

- **Dichotomous scoring** – the concept of scoring the results, that allows users to get either the full amount of points or zero in a case of any mistake;
- **Partial scoring** – the concept of scoring the results in a way that allows users to obtain some points for a question, which they answered only partially correct;
- **Difficulty** – a difficulty weight of the question in the questionnaire in the interval \([0, 1]\). The difficulty can be determined automatically and dynamically based on prior scoring. In our implementation, for example, difficulty is dynamically updated after one student provided an answer, according with the formula:

\[
\text{d} = f(d') = (d' \cdot (d_{\text{max}} - 1)) + 1
\]

where \(d\) is the difficulty, that the question was expected to have, \(d'\) is the difficulty, that the question had at the moment, when the \(p\)-th attempt was made. After the difficulty is determined, it is scaled to the interval \([1, d_{\text{max}}]\), where \(d_{\text{max}}\) is the maximal difficulty, that a question can have.

- **Guessing level** – the theoretical probability to guess the correct answer from the list of options. In partial scoring, we determine the guessing level as the probability to obtain more than zero points.
- **Basic question points** – an absolute value of points for the correctly checked options or the percentage of correctly checked options within all correct options. Basic points = \(f(d)\).

- **Penalty** – the value, that should be deducted from the basic points due to the logic of the applied algorithm. In our approach we propose, that penalty should be only deducted, when user checks more options, than the number of correct ones.

\[
\text{Total question score} = f(p, s).
\]

3 Related work

There are several existing platforms, that use multiple-mark type of questions as well as several approaches to score them. We collected such approaches
to describe, discuss and compare them. Existing approaches for scoring the multiple-mark questions implement four base concepts. In the section we describe the basic ideas, advantages and disadvantages of these concepts.

### 3.1 Dichotomous scoring

This method is often used in paper-based questionnaires, where the good quality of questionnaires allows teachers to be more strict when scoring results. In the case of choosing a wrong option indicates that a student hopes to guess the correct response as she does not know the material behind the question well. In e-based learning the quality of questionnaires is not perfect, especially in the systems with collaborative authoring. That is why the dichotomous scoring can punish the learners for the teachers mistakes too much. As the aim of questionnaires is not only to score the results, but to catch the gaps of knowledge, the scoring of partially correct responses shows the actual knowledge of the student better. Also, dichotomous scoring does not show the accurate progress of the student. However, when dealing with multiple-mark questions dichotomous scoring almost excludes the possibility of guessing, that is why we use it as a standard of reference when evaluating our approach with real users.

### 3.2 Morgan algorithm

One of the historically first methods for scoring the MMQs was described in 1979 by Morgan (Morgan, 1979). In the accordance to the method, the scores are determined by the following algorithm:

1. for each option chosen which the setter also considers correct, the student scores +1.
2. for each option chosen which the setter considers to be incorrect, the student scores -1.
3. for each option not chosen no score, positive or negative, is recorded regardless of whether the setter considers the response to be correct or incorrect.

The algorithm can be improved by changing the constant 1 to dynamically determined amount of points:

1. for each option chosen which the setter also considers correct, the student scores \((p_{\text{max}}/n)\), where \(n\) is a number of correct options
2. for each option chosen which the setter considers to be incorrect, the student scores \(-(p_{\text{max}}/k)\), where \(k\) is a number of distractors.

We use this improved algorithm for our experiments. However, the experiments show a large dependence between number of options (correct and incorrect) and amount of penalty, that indicates the skewness of the method (see Section 5.1).

### 3.3 MTF scoring

Multiple-mark questions can be scored with the approaches developed for multiple true-false items. The base approach to score the MTF items is to determine, how close is the student response to the correct one. Tsai (Tsai and Suen, 1993) evaluated six different implementations of the approach. Later his findings were confirmed by Itten (Itten and Krebs, 1997). Although both researches found partial crediting to be superior to dichotomous scoring in a case of MTFs, they do not consider any of the algorithms to be preferable. This fact allows us to use the most base of them for our experiments.

All the MTF scoring algorithms imply that any item has \(n\) options and a fully correct response is awarded with full amount of points \(p_{\text{max}}\). If the user did not mark a correct option or marked a distractor, she is deducted with the penalty \(s = p_{\text{max}}/n\) points. Thus a student receives points for not-choosing a distractor as well as for choosing a correct option. This point does not fit perfect to multiple-mark questions because of the differences between two types (Pomplun and Omar, 1997; Cronbach, 1941; Frisbie, 1992). Our experiments (see Section 5.1) confirm the studies and show the skewness of the concept when deal with MMQs. The main problem of the MTF scoring method, when applied to MMQs, is that a user obtains points, even if she did not chose any options. Although the problem can be solved by creating an additional rule, the experiments show the further problems of the algorithm, when used for MMQ items.

### 3.4 Ripkey algorithm

Ripkey (Ripkey and Case, 1996) suggested a simple partial crediting algorithm, that we named by the author. In the approach a fraction of one point depending on the total number of correct options is awarded for each correct option identified. The approach assumes no point deduction for wrong choices, but items with more options chosen than allowed are awarded zero points.

The Ripkey’s research showed promising results in a real-life evaluation. However, later researches (e.g. Bauer (Bauer et al., 2011)) notice the limitations of the Ripkey’s study. The main issue in the Ripkey algorithm is the not well-balanced penalty. Our experi-
ments show that in many cases the algorithm penalizes so severely, that learners could consider it to be the dichotomous scoring. We aim to improve the Ripkey’s algorithm by adding the mathematical approach for evaluating the size of penalty.

4 Balanced scoring method for MMQs

4.1 Concepts

As shown above, existing approaches do not solve the problem of scoring MMQs perfectly. Our concept is based on the assumption, that scoring can be based on the guessing level of the question. Thus, when a student marks all possible options, she increases the guessing level up to 1. In this case the student should obtain either the full amount of points (if all the options are considered to be correct by the teacher), or zero, if the question has at least one distractor. However, if a student did not mark any option, the score should be always zero, as we assume that all the questions have at least one correct option. Thus, the task is to find the correctness percentage of the response and decrease it with a penalty, if the guessing level was artificially increased by marking too many options.

Questions have the native level of guessing, and we propose to deduct the penalty only if after the student’s response the guessing level increases. In other words, we determine the penalty only when a student marks more options, than the number of correct ones.

4.2 Mathematical model

In this section we present the mathematical model as well as an algorithm, that can be used for its implementation.

4.2.1 Assumptions and restrictions

We propose to use our approach only in systems, that comply with the following requirements for assessment items:

- all the item’s options have the same weight;
- there is at least one correct option;
- there are no options excluding all other (e.g. "all above are correct")

![Flow chart of the Balanced scoring algorithm.](image-url)
4.2.2 Scoring the basic points

To score the basic points we use the approach, described by Ripkey. Below we present our approach for scoring the penalty.

- \( d \in \mathbb{R}, d \in [1..d_{\text{max}}] \) – difficulty of the current question, for our experiments we set \( d_{\text{max}} = 5 \)
- \( C \subseteq A \) – set of the correct options \( c_i \) for the current question, where \( A \) – set of the options \( a_j \) for the current question,
- \( c_{\text{max}} = |C|, c_{\text{max}} \in \mathbb{N} \) – number of correct options for the current question
- \( C_{\text{ch}} \) – set of the correctly checked options
- \( c_{\text{ch}} = |C_{\text{ch}}|, c_{\text{ch}} \in [0, c_{\text{max}}] \) – number of correctly checked options for the current question
- \( p_{\text{max}} = f(d) = d \cdot K_{\text{points}} \) – maximal possible points for the current question, in our system we set \( K_{\text{points}} = 1 \)
- \( p_c \) – points for the correctly checked option \( c \). As we assume all the correct options have the equal weight,

\[
\forall c \in C_{\text{ch}} | p_c = \frac{p_{\text{max}}}{c_{\text{max}}}
\]

- \( p \in \mathbb{R} \land p \in [0, p_{\text{max}}] \) – the basic points for the current question,

\[
p = \sum_{c \in C_{\text{ch}}} p_c = \frac{p_{\text{max}}}{c_{\text{max}}}
\]

4.2.3 Scoring the penalty

Below we present our approach for scoring the penalty. We use the following designations:

- \( a_{\text{max}} \in \mathbb{N}, a_{\text{max}} = |A| \) – number of options \( a \in A \)
- \( Ch \subseteq A \) – set of checked options
- \( ch = |Ch|, ch \in \mathbb{N}, ch \in [0, a_{\text{max}}] \) – number of checked options for the current question
- \( b \in \mathbb{R}, b \in [0, 1] \) – basic level of guessing for the current question,

\[
b = \frac{c_{\text{max}}}{a_{\text{max}}}
\]

- \( n \in \mathbb{R}, n \in [b, 1] \) – measure, that shows the possibility, that user tries to guess the correct response by choosing too much options; we do not evaluate it in the cases, when \( n = b \),

\[
n = \frac{ch}{a_{\text{max}}}
\]

- \( s \) – penalty for the guessing,

\[
s = n - b \Rightarrow s \in [0, 1 - b]
\]

- \( s_k \in [0, p_{\text{max}}] \) – the penalty, mapped to the maximal possible points.

A mapping function is calculated as follows:

\[
f : s_k \rightarrow s
\]

Given, \( s_k \in [0, p_{\text{max}}] \) and \( s \in [0, 1 - b] \), then

\[
f : s_k \rightarrow s = f : [0, 1 - b] \rightarrow [0, p_{\text{max}}] \Rightarrow
\]

\[
s_k = f(s) = s \cdot \frac{p_{\text{max}}}{1 - b} = (n - b) \cdot \frac{p_{\text{max}}}{1 - b}
\]

4.2.4 Scoring the total question score

The absolute score for the question is trivially determined as

\[
T = f(p, s_k) = p - s_k
\]

The percentage representation of the total score is determined as follows:

\[
T\% = \frac{p - s_k}{p_{\text{max}}} \times 100\%
\]

5 Evaluation

5.1 Synthetic experiments

In the subsection we describe our experiments with synthetic data and compare the behavior of different methods. For shorter presentation, we use the following reductions:

- Dich. – dichotomous scoring;
- Balanced – the proposed balanced scoring method

We consider all the questions to have the difficulty \( d = 1 \), then the maximal possible points \( p_{\text{max}} = 1 \) as well.

Example 1 (Case: 5 options, 2 correct, 5 marked). In the case the student chose all the options and should obtain zero points. However, we see that MTF method does not recognize this type of guessing and considers the questions to be answered partially correct, awarding the points for two correct options, that were marked.

Example 2 (Case: 5 options, 2 correct, 0 marked). The situation is opposite to the previous: in the case the student chose none of the options. As we assume that question must have at least one correct option, in case of not choosing any options a student also should obtain zero points. However, we see that MTF method...
awards the points for three distractors, that were not marked. Although the situation is absurd, we faced it within real learning platforms, for example within several on-line courses of the Stanford University.

Two examples below are trivial and the problem could be solved by adding the rules. However, the MTF scoring also suffers from skewness, when applied to MMQs, as it is shown below.

**Example 3** (Case: 6 options, 2 correct, 1 correct marked). This case proves, that the MTF method has a dependency from a number of correct and incorrect options. Thus, in a case of 6 options two of which are correct, a student is awarded 0.833 points for choosing only one correct option. In a case of 5 options two of which are correct, she would be awarded 0.80 points for the same. Moreover, if she choose only one incorrect option in a case of 6 alternatives, she obtains 0.5 points; in a case of 5 options she will be awarded 0.4 for the same.

Thus, our experiments prove, that multiple-mark questions can not be scored properly with the algorithms, developed for multiple true-false items. Moreover, a teacher should be careful when creating multiple true-false questions and create them in such a manner, that not-choosing a distractor deserves awarding. However, the MTF scoring is the only existing approach of partial scoring that can be used in a case, when a question does not have any correct options.

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Table 1: Comparison of the proposed approach with other existing approaches

<table>
<thead>
<tr>
<th>Dich.</th>
<th>MTF</th>
<th>Morgan</th>
<th>Ripkey</th>
<th>Balanced</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>0.4</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</table>

Table 2: Comparison of the proposed approach with other existing approaches

<table>
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<tr>
<th>Dich.</th>
<th>MTF</th>
<th>Morgan</th>
<th>Ripkey</th>
<th>Balanced</th>
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<tbody>
<tr>
<td>0</td>
<td>0.6</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</table>

Table 3: Comparison of the proposed approach with other existing approaches

<table>
<thead>
<tr>
<th>Dich.</th>
<th>MTF</th>
<th>Morgan</th>
<th>Ripkey</th>
<th>Balanced</th>
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<tbody>
<tr>
<td>0</td>
<td>0.83</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
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</table>

Table 4: Comparison of the proposed approach with other existing approaches

<table>
<thead>
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<th>Dich.</th>
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<th>Morgan</th>
<th>Ripkey</th>
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<tr>
<td>0</td>
<td>0.5</td>
<td>0</td>
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2http://online.stanford.edu/courses

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Example 4 (Case: 4 options, 2 correct, 1 correct and 1 incorrect marked). This case illustrates the issues of using the Morgan algorithm. The Morgan algorithm deducts penalties for choosing the incorrect option, as well as the proposed approach. There are two main issues:

- Does the response deserve penalty?
- If deserves, how big the penalty should be?

In that case we are facing the situation, that penalty has the same size, as the basic points, and the student is awarded zero. We consider the penalty to be needlessly
high, especially because the penalty depends on the number of incorrect options. Thus, if the question has 3 incorrect options, choosing one of them would be fined on 0.33, and in case of 2 incorrect options, the penalty is 0.5. After recognizing behavior of the algorithm, students will mark only the options, they are sure in, because choosing an incorrect one may cost them a full amount of points, they collected with correct options.

The next two examples show mainly the differences between the proposed approach and Ripkey algorithm. Namely, we show the situations, when Ripkey algorithm awards zero points, while we consider that it should award more.

**Example 5** (Case: 4 options, 2 correct, 2 correct and 1 incorrect marked). *In this case the student chose more options, than the number of correct ones, and according to the Ripkey, the answer should be awarded zero. Our claim is, that until the student have not chosen all the options, she could have some points. However, choosing three of four options could mean a try of guessing. Although in this case the student gets the full amount of basic points, she is fined on a half of them.*

**Example 6** (Case: 5 options, 2 correct, 2 correct and 1 incorrect marked). *The example shows the disadvantage of the Ripkey algorithm more clear. It is not clear for the student, why she was awarded zero points, as she did not try to guess and answered partially correct.*

**Example 7** (Case: 5 options, 3 correct, 2 correct and 1 incorrect marked). *In that case balanced scoring and Ripkey algorithms behave the same, as none of them deducts a penalty.*
For evaluation of our algorithm we used a lecture series on “Business Information Systems”. We chose this course since it comprises a large number of definitions and descriptions, which are well suited for the creation of MMQs. In total we have created 130 questions. A course of 30 students was offered to prepare for the final examination using SlideWiki. Overall, the students made 287 attempts to complete the questionnaire and we collected all their answers (also unfinished assessments) for the evaluation. After collecting the answers, we implemented all discussed algorithms to score and compare the results, in particular with regard to the ranking and the mean score. The results are summarized in Figure 3.

The study aimed to investigate three aspects of the proposed approach:

- How severe does the balanced scoring approach penalize?
- How does balanced scoring differ from Dichotomous scoring?
- How clear were the results scored by the proposed approach for the students?

We answer the first question by comparing the scores calculated using all discussed algorithms for the same questionnaire (see Figure 3, upper part). These two diagrams show, that on average the balanced scoring approach penalizes more severely than MTF scoring and less severely than other discussed approaches. Thus, the users study confirms the findings of our previous synthetic experiment.

We answer the second question by comparing the difference in student ranking. We rank all assessments based on the individual scores. That is, assessments with higher scores rank higher than assessments with lower scores and equal scores result in the same ranking. We compare the rankings of other approaches with the rankings calculated using the dichotomous scoring, since we consider the dichotomous scoring to be the ranking reference. The two lower diagrams in Figure 3 show the results of this evaluation. They show, that the ranking of the balanced scoring approach is the closest to the dichotomous ranking when compared to the other algorithms.

After the end of semester we asked the participants to answer the third question. They were offered to evaluate clarity of the results on a five-point scale from “very clear” to “very unclear”. We have collected nine responses, two of them were “neutral”, four – “clear” and three – “very clear”. This confirms that the results obtained by the balanced scoring method are easy to understand for students.
6 Conclusions

In the paper we evaluate the existing approaches for scoring the multiple-mark questions and propose a new one. The proposed approach has a list of restrictions, however it has advantages when compare with the discussed approaches. One of the main advantages is its clearness for the students, that was proven by the user evaluation. Also, our approach is based on the mathematical model, it does not suffer from the skewness, as it has the same formula for all cases. At the same time, the proposed approach recognizes the attempts to guess the correct answer, for example choosing all the possible options. When compare with the existing approaches, the advantages of the proposed algorithm could be summarized as follows:

- The approach allows to score both multiple-mark and conventional multiple-choice questions.
- The approach is based on the partial scoring concept.
- The algorithm can be easily implemented, it is pure mathematical.
- The score does not highly depend on the amount of correct and incorrect options.
- The value of the penalty is in balance with the possibility, that the student is trying to guess.
- Due to the balance, the results are clear for the students.

However, we suppose our algorithm to be optional together with other discussed approaches. This is due to the fact, that teachers create questions in their own manner and should be able to choose an appropriate method to score the results. Also, the different situations require different levels of severity, and the proposed approach might be too lenient.

REFERENCES


